Carlson-Goldman modes in the color superconducting phase of dense QCD

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We predict the existence of the gapless Carlson-Goldman modes in the color-flavor locked color superconducting phase of dense QCD. These modes resemble Nambu-Goldstone modes of the superconducting phase below the critical temperature where the Anderson-Higgs mechanism takes place. These modes exist in the broken phase in the vicinity of the critical line. Their presence does not eliminate the Meissner effect. The effect of Landau damping on the width of the Carlson-Goldman modes is discussed.

11.15.Ex, 12.38.Aw, 21.65.+f

The Anderson-Higgs mechanism is one of the most fundamental notions of modern theoretical physics, having many important applications in particle and condensed matter physics. The Standard Model of particle physics, which has already withstood numerous experimental tests, crucially incorporates the Higgs mechanism as one of the most essential ingredients. Similarly, the theory of the low and high temperature superconductivity would not be complete without the Anderson mechanism.

About a three decades ago, an unusual and a rather surprising "propagating order parameter collective modes" were discovered by Carlson and Goldman [1] (see also, Ref. [2–4]). One of the most intriguing interpretation connects such modes with a revival of the Nambu-Goldstone (NG) bosons in the superconducting phase where the Anderson-Higgs mechanism should commonly take place. It was argued that an interplay of two effects, screening and Landau damping, is crucial for the existence of the CG modes [3,4]. These modes can only appear in the vicinity of the critical temperature (in the broken phase), where a large number of thermally excited quasiparticles leads to partial screening of the Coulomb interaction, and the Anderson-Higgs mechanism becomes inefficient. At the same time, the quasiparticles induce Landau damping which usually makes the CG modes overdamped in clean systems. To suppress such an effect and make the CG modes observable, one should consider dirty systems in which quasiparticle scatterings on impurities tend to reduce Landau damping.

In the two fluid description, the CG modes are related to oscillations of the superfluid and the normal component in opposite directions [5]. The local charge density remains zero in such oscillations, providing favorable conditions for gapless modes, in contrary to the widespread belief that the plasmons are the only collective modes in charged systems.

In this Letter we predict the existence of the Carlson-Goldman (CG) modes in dense quark matter. It has been known for a long time that quark matter at a sufficiently high baryon density should reveal color superconductivity [6,7]. Recent developments [8,9] added a hope that a color superconducting phase may exist in compact stars, and its signatures could possibly be detected [10].

We shall consider the color-flavor locked (CFL) phase of dense QCD with three massless flavors of quarks (up, down and strange). In this phase, the original $SU(3)_C \times SU(3)_R \times SU(3)_L$ symmetry of the QCD action breaks down to $SU(3)_{C+L+R}$ [11]. Out of total sixteen (would be) NG bosons, eight (ϕ^A, $A = 1,\ldots,8$) are removed from the physical spectrum by the Higgs mechanism, providing masses to eight gluons. The other eight NG bosons (π^A) show up as an octet of pseudoscalars. In addition, the global baryon number symmetry as well as the approximate $U(1)_A$ symmetry also get broken. This gives an extra NG boson and a pseudo-NG boson.

The order parameter consists of a color antitriplet, flavor antitriplet $(\bar{3},\bar{3})$ and a color sextet, flavor sextet $(6,6)$ contributions [12]. The dominant contribution to the order parameter is $(\bar{3},\bar{3})$, but a non-zero, although small, admixture of $(6,6)$ also appears [11,13,14]. The value of the order parameter at zero temperature was estimated using phenomenological [8,9], as well as microscopic models [15–18]. While the exact value remains uncertain, it can be as large as 100 MeV. Without loosing generality, here we consider a pure $(\bar{3},\bar{3})$ order parameter, so that the quark gaps in the octet and singlet channel are related as follows $\Delta_1 \approx -2\Delta_8 = -2|\Delta_T|$ at any finite temperature. As we shall see below, the presence of two distinct (octet and singlet) quark gaps in the CFL phase leads to a partial suppression of Landau damping, and, as a result, the CG modes could show up even in a clean color superconductor.

Our current analysis mostly deals with the collective modes, related to the gluon field. It is natural, then, to start our consideration with the effective action, obtained by integrating out the quark degrees of freedom. Irrespective of a specific model, the corresponding Lagrangian density should read

$$\mathcal{L} = -\frac{1}{2} A^{A,\mu}_{-q} i \left[\mathcal{D}^{(0)}(q)\right]^{-1}_{\mu\nu} A^{A,\nu}_{q} - \frac{1}{2}\left[A^{A,\mu}_{-q} - iq^{\mu}\phi^{A}_{-q}\right]\Pi_{\mu\nu}(q)\left[A^{A,\nu}_{q} + iq^{\nu}\phi^{A}_{q}\right] + \ldots, \quad (1)$$

where ellipsis denote the interactions terms. The presence of the phase field octet ϕ^A_q is very important for

preserving the gauge invariance of the model. Under a gauge transformation, the phase ϕ_q^A in Eq. (1) is shifted so that it would exactly compensate the transformation of the gluon field. (Here we consider only infinitesimally small gauge transformations. In general, the gauge transformation of ϕ_q^A field is not a simple shift. However, it is always true that its transformation exactly compensates the transformation of the gluon field.)

In the simplest approximation, the polarization tensor $\Pi_{\mu\nu}$ in Eq. (1) is given by a one-loop quark diagram. Its formal expression was presented in Ref. [19] (also, the zero temperature limit was considered in Ref. [20], and some other limits will be presented in Ref. [21]). Let us discuss the general properties of this tensor.

We start by pointing out that the one-loop polarization tensor $\Pi_{\mu\nu}$ in Eq. (1) obtained by integrating out the quark degrees of freedom is *not* transverse in general. This is directly related to the Higgs effect in dense QCD. The latter implies the presence of the composites with the quantum numbers of the (would be) NG bosons in all nonunitary gauges [22]. In the problem at hand, in particular, this means that the complete expression for the polarization tensor should necessarily contain an additional contribution coming from integrating out the (would be) NG bosons [denoted by ϕ_q^A fields in Eq. (1)]. Having said this, we should note right away that the one-loop expression for $\Pi_{\mu\nu}$ *is* transverse in the normal phase of the quark matter (above T_c) where there are no (would be) NG bosons.

The longitudinal part of the one-loop polarization tensor $\Pi_{\mu\nu}$ has a physical meaning in the Higgs phase of dense QCD. It was shown in Ref. [20] that such a longitudinal part contains the information about the propagators of the pseudoscalar NG bosons, and, thus, determines the corresponding dispersion relations. Strictly speaking, of course, the properties of the NG bosons are related to the "axial" polarization tensor. In the model at hand, the two quantities are identical to the leading order, and we could safely interchange them.

The action in Eq. (1) is a starting point in our analysis. In order to construct the current-current correlation function, we should add the classical external gauge fields to the action in Eq. (1) and integrate out all quantum fields. After doing so, we obtain the generating functional. The correlation function, then, is given by the second order derivative with respect to the external gauge field. In momentum space, the result reads [21]

$$\langle j_\mu^A j_\nu^B \rangle_q = \delta^{AB} \left[\frac{q^2 \Pi_1}{q^2+\Pi_1} O_{\mu\nu}^{(1)}(q) + \frac{q^2\left[\Pi_2\Pi_3+(\Pi_4)^2\right]}{(q^2+\Pi_2)\Pi_3+(\Pi_4)^2} O_{\mu\nu}^{(2)}(q) \right]. \quad (2)$$

Here Π_i $(i=1,\dots 4)$ are the component functions of the one-loop polarization tensor, introduced as follows:

$$\Pi_{\mu\nu}(q) = \Pi_1 O_{\mu\nu}^{(1)} + \Pi_2 O_{\mu\nu}^{(2)} + \Pi_3 O_{\mu\nu}^{(3)} + \Pi_4 O_{\mu\lambda}^{(4)}, \quad (3)$$

and

$$O_{\mu\nu}^{(1)}(q) = g_{\mu\nu} - u_\mu u_\nu + \frac{\vec{q}_\mu \vec{q}_\nu}{|\vec{q}|^2}, \quad (4)$$

$$O_{\mu\nu}^{(2)}(q) = u_\mu u_\nu - \frac{\vec{q}_\mu \vec{q}_\nu}{|\vec{q}|^2} - \frac{q_\mu q_\nu}{q^2}, \quad (5)$$

$$O_{\mu\nu}^{(3)}(q) = \frac{q_\mu q_\nu}{q^2}, \quad (6)$$

$$O_{\mu\nu}^{(4)}(q) = O_{\mu\lambda}^{(2)} u^\lambda \frac{q_\nu}{|\vec{q}|} + \frac{q_\mu}{|\vec{q}|} u^\lambda O_{\lambda\nu}^{(2)}, \quad (7)$$

are the three projectors of different types of gluon modes ("magnetic", "electric", and unphysical "longitudinal") [16], and one intervening operator, respectively. By definition, $u_\mu = (1,0,0,0)$ and $\vec{q}_\mu = q_\mu - (u\cdot q)u_\mu$.

The current-current correlation function in Eq. (2) is explicitly transverse. Similarly, the expression for the polarization tensor that includes the contribution of the would be NG bosons is also transverse. And, the corresponding gluon propagator reads

$$i\mathcal{D}_{\mu\nu}^{(g)}(q) = \frac{1}{q^2+\Pi_1} O_{\mu\nu}^{(1)} + \frac{\Pi_3}{(q^2+\Pi_2)\Pi_3+(\Pi_4)^2} O_{\mu\nu}^{(2)} + \frac{1}{\lambda q^2} O_{\mu\nu}^{(3)}. \quad (8)$$

The poles of the correlation function (2) and the gluon propagator (8) define the spectrum (as well as screening effects) of the magnetic and electric type collective excitations,

$$q^2 + \Pi_1(q) = 0, \quad \text{"magnetic"}, \quad (9)$$

$$\left[q^2+\Pi_2(q)\right]\Pi_3(q) + \left[\Pi_4(q)\right]^2 = 0, \quad \text{"electric"}. \quad (10)$$

To proceed with the analysis, we need to know the explicit expression for the polarization tensor. The general finite temperature result of Ref. [19] is rather complicated. We notice, however, that some limiting cases are easily tractable [21]. Here we pay a special attention to the nearcritical region ($T \lesssim T_c$ and $|\Delta_T|/T \ll 1$), where the CG modes are expected to appear.

Before presenting our main results, we would like to mention that the properties of CG modes are extremely sensitive to the Landau damping effects. A clear signal of the CG modes usually appears only in dirty samples where Landau damping is rather inefficient [1]. In the clean (no impurities) limit, on the other hand, Landau damping is so strong that CG modes may become unobservable [4].

Here we assume that quark matter in the CFL phase is a clean system. This may or may not be true, because there could exist some natural impurities in the vicinity of the critical point. However, if we show (as we actually do) that CG modes exist in the clean limit where Landau damping is strongest, then adding impurities into consideration can only improve the quality of the CG modes.

In the nearcritical region, the calculation of the one-loop polarization tensor [see Refs. [19,21] for the general

representation] could be done approximately as an expansion in powers of $|\Delta_T|/T$. The result reads

$$\Pi_1 \simeq -\omega_p^2 \left[\frac{7\zeta(3)|\Delta_T|^2}{12\pi^2 T^2} + \frac{7}{3}v^2 - \frac{7i\pi}{12}v + \frac{\pi v|\Delta_T|}{96T}\left(5\pi + 33i + 10i\ln\frac{v}{2}\right)\right], \quad (11)$$

$$\Pi_2 \simeq \omega_p^2 \left[-3 + \frac{21\zeta(3)|\Delta_T|^2}{4\pi^2 T^2} + 4v^2 - \frac{7i\pi}{6}v + \frac{\pi v|\Delta_T|}{48T}\left(9\pi + 28i + 18i\ln\frac{v}{2}\right)\right], \quad (12)$$

$$\Pi_3 \simeq \omega_p^2 \left[-\frac{7\zeta(3)|\Delta_T|^2}{12\pi^2 T^2} + \frac{2}{3}v^2 + \frac{5i\pi|\Delta_T|}{48T}v\right], \quad (13)$$

$$\Pi_4 \simeq \frac{2}{3}\omega_p^2 v, \quad (14)$$

where $\omega_p = g_s\mu/\sqrt{2}\pi$ is the plasma frequency, T is the temperature, $|\Delta_T|$ is the value of the gap at this given temperature, and $v \equiv q_0/|\vec{q}|$. In our derivation, we treated both $|\Delta_T|/T$ and v as small parameters of the same order. All cubic and higher order terms has been dropped. We also restricted ourselves to the low-energy region $|q_0| \ll |\Delta_T|$. Now, by making use of the above explicit expressions for the polarization tensor components, from Eq. (10) we derive the approximate dispersion relation of the electric type modes:

$$v_{cg}^2 + i\frac{45\pi|\Delta_T|}{224T}v_{cg} - \frac{9\zeta(3)|\Delta_T|^2}{8\pi^2 T^2} = 0, \quad (15)$$

The solution to the dispersion equation (15) reads

$$q_0 = \frac{|\Delta_T|}{T}(\pm x^* - iy^*)|\vec{q}|, \quad (16)$$

where $x^* \approx 0.193$ and $y^* \approx 0.316$. This solution corresponds to the gapless CG mode. The width of this mode is quite large, but this is not surprising since the clean limit of quark matter is considered. We have also derived an analytical expression which does not assume the smallness of v_{cg}, and which includes all the corrections up to order $(\Delta_T/T)^2$. Because of the complicated structure, see Ref. [21] for details, we do not present the corresponding equation here. Instead, we describe the numerical solution in words. We found that the ratio of the width, $\Gamma = -2Im(v_{cg})|\vec{q}|$, to the energy of the CG mode, $\varepsilon_q = Re(v_{cg})|\vec{q}|$, increases when the temperature of the system goes away from the critical point. At temperature $T^* \approx 0.986T_c$, this ratio formally goes to infinity. This value of T^* could serve as an estimate of the temperature at which the CG mode disappears. Of course, such an estimate is rather crude because our approximations break before the temperature T^* is reached.

Because of the large imaginary part in Eq. (16), one might conclude that the CG modes cannot not be observable in the clean CFL phase of quark matter. By calculating the spectral density of the electric type gluons in the close vicinity of the critical temperature, we see that such a conclusion is not quite correct. As is shown in Fig. 1, there is a well defined peak in the spectral density which scales with the momentum in accordance with a linear law. To plot the figure, we used the standard Bardeen-Cooper-Schrieffer (BCS) dependence of the value of the gap on temperature, obtained from the following implicit expression:

$$\ln\frac{\pi T_c}{e^\gamma|\Delta_T|} = \int_0^\infty \frac{d\epsilon\left(1 - \tanh\frac{1}{2T}\sqrt{\epsilon^2 + |\Delta_T|^2}\right)}{\sqrt{\epsilon^2 + |\Delta_T|^2}}, \quad (17)$$

where $\gamma \approx 0.567$ is the Euler constant. As was shown in Ref. [18], such a dependence remains adequate in the case of a color superconductor.

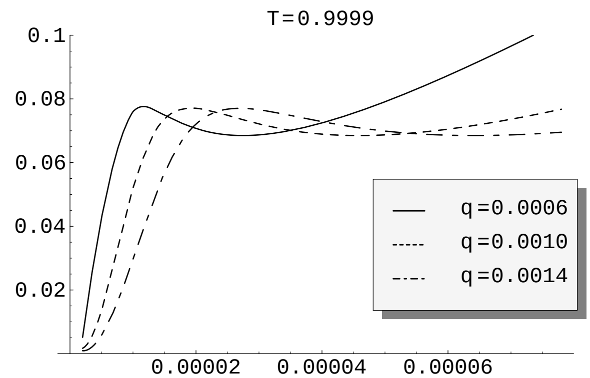

FIG. 1. The spectral density of the electric modes at $T = 0.9999T_c$ (corresponding to $|\Delta_T| \simeq 0.031T_c$) as a function of energy. Everything is measured in units of T_c.

It has to be emphasized that the results in Eq. (16) and Fig.1 are obtained for a clean system where Landau damping effects have their full strength. Therefore, it should have been expected that the CG modes are overdamped. However, our analysis shows that the presence of two different types of quarks with nonequal gaps in the CFL phase plays an important role in partial suppression of Landau damping. We checked that if all quarks had the same value of the gap in their spectrum (as in ordinary metals), the ratio of the imaginary and real parts in the solution [$v \simeq -14i\zeta(3)|\Delta_T|/3\pi^3 T$ which is pure imaginary in the approximation considered] would be much larger than 1, meaning that the CG modes would be unobservable in clean systems.

Now, let us discuss the Meissner effect near T_c. This can be done by examining the response of the quark system to an external static magnetic field. So, we consider the magnetic Π_1 component of the polarization tensor in the limit $|q_0| \ll |\vec{q}| \to 0$. By substituting $v = 0$ on the right hand side of Eq. (11), we obtain

$$\Pi_1 \simeq -\frac{7\zeta(3)|\Delta_T|^2}{12\pi^2 T^2}\omega_p^2. \quad (18)$$

Since the right hand side is nonzero, it means that a static magnetic field is expelled from the bulk of a supercon-

ductor. Thus, the conventional Meissner effect is unaffected by the presence of the CG modes. We would like to note that the scale, given by $\omega_p|\Delta_T|/T$ in Eq. (18), is not directly related to the penetration depth of the magnetic field. In order to determine the actual depth, we would need to know the dependence of the Π_1 component for a relatively wide range of spatial momenta, $0 < |\vec{q}| \lesssim \omega_p|\Delta_T|/T$. Without doing the calculation here, we mention that the penetration depth should be of order $\lambda_P \simeq (|\Delta_T|\omega_p^2)^{-1/3}$ (the Pippard penetration depth) [21].

In passing, we remind that, because of spontaneous breaking of chiral symmetry, there are real NG bosons in the CFL phase of a color superconductor. Unlike the scalar CG modes, the NG bosons are pseudoscalars. According to Ref. [20], their dispersion relation is determined by the equation: $q^\mu \Pi_{\mu\nu} q^\nu \equiv q^2 \Pi_3 = 0$. In the nearcritical region, as follows from Eq. (13), this is equivalent to

$$v_{ng}^2 + i\frac{5\pi|\Delta_T|}{32T} v_{ng} - \frac{7\zeta(3)|\Delta_T|^2}{8\pi^2 T^2} = 0, \qquad (19)$$

which is qualitatively the same as Eq. (15), and it has a solution with similar properties. In fact, the dispersion relation for the NG bosons is given by an expression of the same type as that in Eq. (16), but with $x^* \approx 0.215$ and $y^* \approx 0.245$. By comparing the spectrum of the CG modes with the spectrum of the NG bosons, we see that they are quite similar. This observation may provide some indirect justification to interpret the CG modes as "revived" NG bosons. Such an interpretation, however, should be used with a caution.

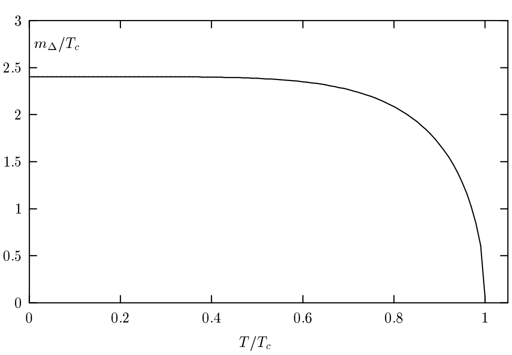

FIG. 2. Mass of the light massive gluon mode as a function of temperature.

For completeness, let us mention that there are two massive gluon modes in the CFL phase [21]. One of them is a plasmon with the mass ω_p, and the other is a relatively light mode with the mass in the range $|\Delta_T| < q_0 < 2|\Delta_T|$. The plasmon has very narrow width, while the other mode is stable. The dependence of the light massive mode on temperature is graphically presented in Fig. 2.

In conclusion, by making use of an explicitly gauge covariant approach, we predict the existence of the gapless CG modes in the CFL phase of cold dense quark matter in the near-critical region (just below T_c) where a considerable density of thermally excited quasiparticles is present. It is important to mention that the presence of the CG modes coexists with the usual Meissner effect, i.e., an external static magnetic field is expelled from the bulk of a superconductor. The existence of the Meissner effect is a clear signature that the system remains in the symmetry broken (superconducting) phase.

In the case of the CFL phase, as we showed, the CG modes appear even in the clean limit. Despite the sizable width, their traces can be observed in the spectral density of the electric gluons. Taking the effect of impurities into account should, in general, make the CG modes more pronounced [4]. In realistic systems such as compact stars, natural impurities of different nature could further improve the quality of the gapless CG modes.

The existence of a gapless scalar CG modes (in addition to the pseudoscalar NG bosons) is a very important property of the color superconducting phase. They may affect thermodynamical as well as transport properties of the system in the nearcritical region. In its turn, this might have a profound effect on the evolution of forming compact stars. The CG modes might also have a consequence on a possible existence of the hypothetical quark-hadron continuity, suggested in Ref. [23]. Indeed, one should notice that, in the hadron phase, there does not seem to exist any low energy excitations with the quantum numbers matching those of the CG modes.

In the future, it would be interesting to generalize our analysis to the so-called $S2C$ phase of dense QCD. In absence of true NG bosons in the $S2C$ phase, the gapless CG modes might play a more important role. Our general observations suggest that Landau damping should have stronger influence in the case of two flavors. At the same time, the presence of massless quarks could lead to widening the range of temperatures where the CG modes exist. To make a more specific prediction, one should study the problem in detail.

Acknowledgments. Authors thank V.A. Miransky and K. Yamawaki for interesting discussions. V.P.G. is grateful to S. Sharapov for fruitful discussions on CG modes in standard superconductors. I.A.S. is grateful to A. Goldman for useful comments. This work is partially supported by Grant-in-Aid of Japan Society for the Promotion of Science (JSPS) #11695030. The work of V.P.G. was also supported by the SCOPES projects 7 IP 062607 and 7UKPJ062150.00/1 of the Swiss NSF. He wishes to acknowledge JSPS for financial support. The work of I.A.S. was supported by the U.S. Department of Energy Grant No. DE-FG02-87ER40328.

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